A case study on the impact of teacher mathematical knowledge on pedagogical practices

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This study explores the implications for mathematical knowledge and pedagogical practices of primary teachers in Papua New Guinea making a shift from a transmission approach to a connectionist approach to teaching mathematics. The research participants were engaged in a professional development program designed to support the teaching and learning of mathematics using a connectionist approach. The data suggest that teacher capacity to adopt a connectionist approach was influenced by their subject knowledge and that their subject knowledge influenced classroom pedagogy.

Introduction

The specific prompt for the research reported here was a teacher professional learning program focusing on approaches to mental calculation in Papua New Guinea. The advantages of an emphasis on teaching mental calculation have been widely reported by researchers (McIntosh, 2004; McIntosh, Reys, & Reys, 1997; Sowder 1990). Changes in mathematics curricula across the world in the last two decades reflect the importance of learning mental calculation.

Separate from, but connected to this, is that the implementation of reform mathematics curricula has implications for pedagogy. It involves a shift in approach from “passive to active learning, and from transmissive to connected and challenging teaching” (Sullivan, 2011, p. 25). Research on the implementation of reform curriculum based on principles of constructivism has highlighted the need for teachers to re-examine their roles within the classroom and create a learning community that allows for co-constructed learning (Simon, 1995; Fosnot, 1996).

Implementation of reform pedagogy also has implications for teacher subject knowledge. An approach to teaching mathematics that requires students to think, reason and problem solve using a range of strategies places demands on teacher subject knowledge. Research conducted on the subject knowledge of prospective primary teachers indicates that this is low (Morris, 2001; Hill, Rowan, Rowan, & Ball, 2005). There has been limited research on the subject knowledge of practising teachers.

This study of teachers aiming to transform their teaching practice to implement a new curriculum with an emphasis on mental calculation provides qualitative research on two aspects of teacher knowledge identified by Hill, Ball, and Schilling (2008): subject matter knowledge and pedagogical content knowledge. The aim of this case study was to examine key factors influencing changes teachers make to teaching practice in an effort to teach based on the principles of a connectionist approach (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997). The research participants were engaged in a mathematics professional development program designed to support an approach to teaching based on connectionist principles. Data collected from semi-structured interviews, lesson observations and planning samples provided evidence to suggest that teacher actions were largely influenced by their subject and pedagogical knowledge of mathematics.

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For the purpose of this research, mental calculation is broadly interpreted to involve challenging students with calculations in which they have to work out the answer using known facts, rather than simply recalling facts they have memorized. It is a “vehicle for promoting thinking” (McIntosh, Nohda, Reys, & Reys, 1995, p. 238).

Research Framework

The aim of this case study was to explore key factors influencing changes teachers make to teaching practice in an effort to teach mental calculation based on the principles of a connectionist approach (Askew et al., 1997). The research was informed by a framework shown in Figure 1. The framework presented by Sullivan, Walker, Borcek, and Rennie (2015) implies that teaching practice (classroom actions) is informed by three nodes: teacher subject knowledge of mathematics and pedagogy; teacher disposition; opportunities and constraints. The three nodes of the framework interact with each other. The focus of this research was on how each of these factors inform teaching practice (classroom actions). The findings related to the first node of the framework: teacher subject knowledge of mathematics and pedagogy, form the basis of this report. The central research question for this study was:

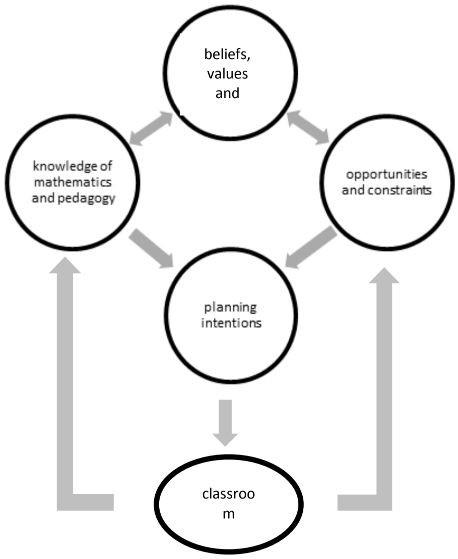

Figure 1 The framework informing the research.

> In what ways does teachers’ mathematical knowledge, especially related to mental calculation, inform their planning and practice?

Teacher knowledge of mathematics and pedagogy is one node of the research framework. The research explored two main aspects of teacher knowledge as proposed by Hill et al. (2008): subject knowledge of mathematics (content knowledge); and pedagogical knowledge. The framework implies that if teachers understand the mathematics, have knowledge about pedagogy for effective teaching and learning of mental calculation, and can predict potential student responses, this will be reflected in teaching practice. In situations where teachers are not confident with their subject knowledge, research suggests that teachers avoid the risk of students asking unplanned questions (Goulding, Rowland, & Barber, 2002).

Teacher dispositions (beliefs, values and attitudes) are another node of the research framework. The framework implies that teaching practice is affected by teacher beliefs about how students learn, their views on mindsets and student performance (Dweck, 2000). For example, research conducted into effective teachers of numeracy (Askew et al., 1997) suggested that teacher questioning was influenced by teacher beliefs. If teachers believed that nearly all students were capable of learning they challenged all students to think, communicate and problem solve. Teachers also have beliefs about what they think constitutes an effective teacher, and their role in the classroom, which influences classroom practice.

The final node of the framework concerns opportunities and constraints. The framework implies that teaching practice can be influenced by potential constraints that teachers anticipate in teaching mental calculation. For example, if teachers anticipate that students will not share explanations for fear of making mistakes in front of their peers they may tend to model strategies. The contexts in which teachers work can also present constraints or opportunities, for example, curriculum documents or polices, parent and student expectations and the classroom environment.

Research design and methodology

The theoretical framework for this research is social constructivism. This suggests that the research on changes to teaching practice are viewed through the individual experiences of the research participants, with the researcher co-constructing understandings through interactions with the participants.

The methodology for this research was qualitative and a case study approach was used to synthesise and report the data. The research aimed to gain further insights into how teacher pedagogical content knowledge, teacher disposition and constraints interact and inform teaching practice. The research was informed by the framework comprised of three nodes representing each of the aforementioned influences on classroom actions.

It is important to provide some description of the context and setting of this research. The research is of significance to the International Education Agency (IEA), a private education system in Papua New Guinea (PNG). The research participants teach at a primary school owned and operated by the IEA. The IEA released a new mathematics curriculum at the beginning of 2015, which recommends an approach to teaching based on the research of effective teaching of numeracy conducted in Australia and the UK.

The research was conducted at an IEA primary school located in the capital city, Port Moresby. The school had been committed to improving the teaching of mathematics over the last two years. The purpose of the research was not to evaluate the professional development program; the aim was to gain some insight into how teacher actions were informed. The research participants were experienced Papua Guinean primary teachers who had participated in the school based professional development programs for mathematics and the learning environment facilitated by IEA advisors.

The focus of the mathematics professional development program was on developing subject knowledge with a focus on concepts of place value and mental calculation strategies; how children learn concepts of place value and mental calculation strategies; pedagogy to deliver quality interactive teaching. The teachers were provided with opportunities to plan with colleagues, observe and co-teach mathematics lessons with IEA advisors.

The school also engaged in professional learning sessions to help establish a positive learning environment. The sessions focussed on ways of establishing a learning culture in which everyone is considered a learner, and taking risks and making mistakes is considered part of the learning process.

Data collection and analysis

The research involved describing and analysing three sources of data collection: semi-structured interviews, lesson observations and planning documents. There were four participants and the issues that needed to be investigated were complex and required collecting detailed information. The time participants could commit to the research was also a factor. Semi-structured interviews were the main data source: data from the observations and planning documents were used to triangulate data collected from the interviews. A semi-structured interview was selected as the most appropriate data source to collect information from participants from a very much oral society where language and the art of storytelling is an important part of the culture. The data were collected mid-term four of the school year; the participants had been teaching the new mathematics curriculum for almost a full school year.

The participants supplied planning documents. The planning documents provided evidence of teacher subject knowledge, indicated by the connections made between mathematics concepts and the problems and activities planned. The planning documents were also useful in providing the researcher with evidence that could be used to interpret how well teachers knew their students (i.e. were the activities appropriately matched to student skills and knowledge).

The analysis of the data was interpretive in that it involved re-reading and examining each source. Participant responses interpreted as significant were highlighted in the interview transcripts and initial codes generated; these decisions were informed by research on the effective teaching of mathematics. Evidence for each participant describing and explaining ways in which each node influenced teacher actions was summarized. The summaries were recorded in a grid and this highlighted some common themes. The emergence of common themes resulted in a decision to report the data through one voice; this was also considered a preferable option for ethical reasons. The data sources were cross referenced and a summary of key evidence from each data source for each participant was recorded on a matrix.

Findings

The data collected provided insight on the extent to which the three nodes of the research framework – teacher subject knowledge of mathematics and pedagogy, teacher disposition opportunities and constraints – informed the teaching of mental calculation. The findings related to the first node of the framework – teacher subject knowledge of mathematics and pedagogy – are the focus of this report. The data suggest that teacher capacity to adopt elements of a connectionist approach to teaching were influenced by their subject knowledge and that their subject knowledge influenced pedagogical practices. Examples from the data were selected because they highlight responses interpreted as significant in answering the research question. For ethical reasons, the four participants are referred to as T1, T2, T3 and T4.

Teacher Knowledge of Mathematics and Pedagogy

Subject Knowledge

All research participants described the challenges they faced in teaching the content of the new mathematics curriculum. The interview data suggest that the participants considered their professional learning to be in its early stages and as a continuous journey. The following excerpts from the interview transcripts suggest that subject knowledge presented a challenge:

> T1 … this is something different, this is something I never learned before. I like maths but what I am learning now has pushed me further, I mean it has actually sharpened my maths skills…
>
> T2 …I'm still learning but it's been a fun experience. A really big learning experience and so challenging.
>
> T1 ...for me it's been a long journey. I am yet to fully understand the math curriculum itself, the big picture.

The excerpts suggest that the teaching of mental calculation strategies presented the participants with the challenge of learning new subject content.

The implementation of the new curriculum was supported with planning documents that linked together the key concepts in a unit and suggested a sequence of learning activities for teachers to develop. Teachers indicated that the planning documents were useful in supporting the development of their subject knowledge:

> T3 …I know where the maths topics are linked because previously I didn't see that, I taught all over the place and didn't know where to start.

The excerpt suggests that teacher knowledge of the connections between aspects of mathematical content was limited and that the provision of support materials was useful in guiding the planning and teaching of mathematics.

The data from the interviews suggest a relationship between subject knowledge and pedagogical choices by teachers:

> T2 …I'm now finding maths lessons more fun, more easy to deliver. Prior it was probably like me being 'this is how you do it, this is how I would like it to be done' so it was more like a teacher directive rather than being that facilitator.

The excerpt suggests that as teachers develop their subject knowledge and become more confident they are in a position to discuss mathematics with students and allow students to make decisions about calculations to solve problems.

Data collected from the interviews suggest that on occasions teacher subject knowledge affected the capacity of the teacher to respond to students during the lesson, as indicated in the following excerpt:

> T1 …Sometimes they get me, they get me by surprise….So I would pause and let them explain a bit how they got their answers for that particular problem and then I would redirect them onto what I have planned for.

The excerpt suggests that on occasions when the teacher was not aware of the potential connections or explanations a student may form in response to a problem, the situation would be managed by adhering to the content of the lesson plan. Another participant explained that although situations were encountered when it may be useful to diverge from the planned path during a lesson it was not something that was done.

> T4 …it's the discipline, I want to stick to what I have planned for.

The participant did not elaborate on the situation; it could be interpreted as an outcome attributed to uncertainty with subject knowledge or accountability for teaching the curriculum content taking precedence over meeting student learning needs.

The data suggest that limited subject knowledge affected teacher capacity to effectively plan differentiated activities, with evidence suggesting that teachers were not aware of the complexity involved in some tasks or clear about steps of progression to challenge students. In some cases, teachers had not made connections between different aspects of mathematics which affected their capacity to guide students in developing a strategy to solve a problem.

Pedagogical Knowledge

All the participants described their own educational experiences, both at school and at teacher training college as being "strictly teacher directed". Through the eyes of a social constructivist this would be viewed as having an impact on classroom actions. The data collected from the teacher interviews suggest that this presented a challenge for the research participants:

> T2 …In the first and second year it was just taught to us the way we were taught by our teachers; strictly teacher directed and basic algorithms you need to know so you will be able to teach the kids well and that sort of thing. So more restricted to the books and 'do this, do that'.

> T3…And coming with that experience and coming into the classroom as a teacher I had that in my head and as I was teaching that was like part of me.

The excerpts suggest that prior to the professional learning sessions the participants emulated the transmission approach of the teachers they experienced during their school years and at teachers' college.

One of the most striking elements of each interview was the participants' description of the students' positive attitude, engagement and growing confidence in mathematics. This was confirmed by the classroom observations. The following excerpt suggests that the change in attitude towards learning mathematics can largely be attributed to classroom pedagogies:

> T2…in the past maths was more lame, that's how I'd describe it. The way I was taught and the way I was teaching before this new maths curriculum, it was too plain and lame and boring for kids. And I guess with the way we delivered it, that was probably one reason why kids really didn't enjoy maths and the concepts were not there and so the poor results in maths were probably part of the way we delivered it and the approaches we had towards kids.

The interview responses suggest that the participant's perception of the purpose of learning mathematics at school was in the process of changing.

> T4…I would say that the traditional ways, though worked for us maybe at that time, they were not that effective. Here when we look at lifelong learners we want the kids to be able to use that maths. Like in the past we wouldn't see maths as something we can use in life but with the maths we are using now, the type of teaching we are doing now it involves a lot of things that they do. So it is based on experience and they see how the maths they learn in class can actually be used…so there's just a lot of change.

The excerpt reinforces pedagogy of mathematics as being more connected in the sense of providing more "meaningful experiences" for students (Jorgensen, Grootenboer, Niesche, & Lerman, 2010, p. 129).

One of the elements recognised as constituting a connectionist approach to teaching mathematics is the use of dialogue to explore understandings (Askew et al., 1997). The participants described the use of questioning and dialogue as common tool in the mathematics lesson:

> T2 … I tend to question the kids more often when they come up with an answer just to see how they were able to reason and find their calculations. And you find through their justifications and reasons you are able to work out whether there is linking from there to there.

> T3 … So to me, I just listen to them and then I ask them again and use the questions to prompt them like: 'What do you think? How did he come up with that answer? Is it correct? Explain.' They then go on to explain and that's how they learn from each other and they know that there are different strategies to come up with an answer.
>
> T4 … A lot of questioning, just to prompt them a bit to get them to be thinking and at the same time listen for what they have to say.

The classroom observations suggest that mathematics lessons were interactive, the lessons were characterized by a high frequency of questions posed by teachers to engage students in class discussion. The students were given opportunities to share calculation methods. The sharing of calculation strategies predominantly involved students showing what they did rather than explaining their reasoning behind the calculation. The classroom dialogue, although effective in maintaining a balance between teacher talk and student talk, reflected an IRF (item-response-feedback) structure. There are various factors which may have influenced the nature of the interactive session, for example, students are in the early stages of learning how to communicate mathematical thinking, teachers and students are learning about the connections between mathematical content and ideas, and time constraints. One participant reflected on the limitations of pedagogical knowledge and raised the issue of questioning skills, describing the need for "good quality questions to ask or prompt" student thinking. This finding echoes the research by Klein (2001) who highlighted the need to develop questioning skills to support classroom discourse.

Conclusions and implications

The aim of this case study was to examine the key factors influencing changes teachers make to teaching practice in an effort to teach mental calculation based on the principles of a connectionist approach (Askew et al., 1997). The research investigated to what extent teacher subject knowledge of mathematics and pedagogy, teacher disposition and opportunities and constraints informed teaching practice. Although the nodes of the research framework interact with each other, the focus of this report was predominantly on the findings for the first node of the framework, teacher subject knowledge of mathematics and pedagogy. The central research question was:

> In what ways does teachers' mathematical knowledge, especially related to mental calculation, inform their planning and practice?

The research participants were engaged in a mathematics professional development program focused on the teaching and learning of mental calculation and professional learning sessions to support establishing a learning community. The research participants had each faced various challenges in teaching the new mathematics curriculum: They were faced with learning new subject matter and making significant changes to pedagogies. The data provided evidence to suggest that teacher decisions and actions were largely influenced by teachers' subject knowledge of mathematics.

The relationship between subject and pedagogical knowledge is complex. The data suggest that teacher capacity to adopt a connectionist approach to teaching was influenced by their subject knowledge, and that their subject knowledge also influenced pedagogical practices. The participants in the study had the foundational structures in place for co-constructed learning to occur. However, the data from the lesson observations suggest that the quality of interactions between the teacher and students, and the use of opportunities to build on interactions between students, was at times limited due to either subject knowledge or pedagogical knowledge or both. The data indicated, though, that teacher practice is changing and developing as the subject knowledge and pedagogical knowledge is developing. This could be interpreted as indicating that limitations with subject knowledge led to situations in which the teacher could not respond flexibly to student responses in the lesson to address the crux of the issue; it did appear that teacher limited knowledge of students (a sub-category of pedagogical knowledge) led to situations where students struggled to achieve the intended learning and the teachers struggled to address this effectively. The participants recognized the need to further develop both subject knowledge and pedagogical practices to facilitate classroom dialogue and support student learning. The participants seemed accurate in their self-reflections about their teaching.

The findings of the research have implications for future professional development for teachers at the school in which the research was conducted and also for the IEA system. They suggest that ongoing support to develop subject knowledge and pedagogical practices is needed to continue to improve the quality of teaching and learning. The findings also support the view advocated by Brodie et al. (2002) that more research is needed on teachers' responding to and developing the ideas of students, including dealing with misconceptions and identifying and intervening in gaps in student learning.

In conclusion, these findings suggest that teacher practice is changing and developing as subject knowledge is developing. The data suggests that teacher capacity to adopt a connectionist approach is influenced by subject knowledge and that subject knowledge influences pedagogical practices.

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